Putting Together the Math Puzzle:
The Investigation of Math Skill Competencies and Related Cognitive Processes

Adam Scheller, Ph.D.
Senior Educational Consultant
Pearson

Objectives

• Describe the developmental building blocks, cognitive processes, and math related skills correlated with the acquisition and competent performance of mathematics.

• Illustrate how practitioners can...
  1. plan an effective assessment and use the data to document under-achievement and disability in mathematics,
  2. conduct a differential determination to identify which specific mathematics-related processes a child experiences, and
  3. link those assessment findings to appropriate interventions.

Learning Mathematics . . .

. . . requires continual integration and coordination of conceptual and procedural knowledge.

(Rittle-Johnson, Siegler, Alibali, 2001)

349 + 23 = ___

Math-Related Cognitive Processes

Working Memory Theory

(Coding Alphanumeric Stimuli in Working Memory) (Berninger, 2007)
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Executive Functions—Switching Set
(Berninger, 2007)

- Visual symbols of numbers (numerals)
- Writing numeric symbols
- Making automatic associations of verbal names with visual numerals
- Accessing math facts in long-term memory (LTM)

Executive Functions—Inhibition
(Berninger, 2007)

- Switching Stimuli Samples
- Inhibition in Working Memory

Cognitive Processes

- Coding in working memory
- Visual-spatial and temporal-sequential aspects of computational operations
- Executive monitoring of math operations
- Reasoning with numbers to solve math problems
- Holding in memory quantitative, visual-spatial, and verbal information while working on the problem

Mathematics Skills and Mathematics-Related Competencies and Processes

Oral Counting
Math Facts and Arithmetic Operations
Math Fact Retrieval
Computational Operations
Conceptual Knowledge
- Place Value
- Part-Whole Relationships
Math Problem Solving
- Representing problems
- Planning steps for problem solution
- Self-monitoring
Let’s review this in the form of a case study.
Referral for Evaluation

- Process of Hypothesis Generation
  - Gather relevant data to form diagnostic and/or strengths and weaknesses hypotheses

- Process of Hypothesis Testing
  - Given hypothesis, begin to systematically test
    - Rule out to rule in...

Flexibility...

- Is key!!!
- Personalize test batteries and be flexible
  - Based on the unique characteristics of their students’ learning.
- and...be flexible
  - By remaining flexible during the assessment process
    - Evaluators are able to continually test hypotheses in shorter “bursts,”
      - Quicker feedback to make decisions
    - Look at strengths and deficits continuously and in conjunction with each other in order to drive the next assessment,
      - Rule in...rule out...
- This maximizes the effectiveness of the time spent on assessment.

Case Example

Introducing Steven

Background Information

- Meet Steven . . . a 12-year-old boy in the 6th grade. He enjoys science and eagerly talks about research that he is conducting on various topics. He appears to enjoy one-on-one time with adults when he is allowed to discuss topics of interest to him.
- Steven is inquisitive and likes to figure out how things work. He is fascinated by clocks and likes to take them apart.

Background Information

- Steven is often impulsive, especially when he sees something that appeals to him. He has trouble following directions, and frequently requires them to be repeated.
- He has been experiencing behavior problems at school. The teacher describes Steven as forgetful and says that he needs frequent reminders to stay on task in the classroom.

Background Information - School

- In school, Steven is...
  - struggling to comprehend grade-level text
  - His performance on writing and math assignments is below that of his peers.
  - His teachers have observed that Steven is often talking when he should be working.
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Background Information - Home

- His parents have...
  - observed similar behaviors at home.
  - They note that Steven often does not remember his chores. When he does remember, he tends to stop before the tasks are completed.
  - He complains about homework and refuses at times to complete written assignments.
- Both parents and teachers describe Steven as a bright young boy who needs to work harder on school work.

Reasons for Referral

- Steven is being assessed to determine if his difficulties with following directions are because of his current cognitive functioning.
- Another goal of the assessment is to determine if any of Steven’s academic challenges are severe enough to require individualized, intensive intervention.
- Why is Steven struggling to master grade-level objectives in math?
  - The discussion of the assessment results will focus on math, but keep in mind the writing concern.

Let’s put together a test battery

- Not a lot of pre-referral test info, pretty general referral questions
- Start with some staples to gain general info
  - WISC-IV
  - WIAT-III

Why is Steven Struggling to Master Grade-Level Objectives in Math?
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WISC-IV

Ability—WISC-IV

<table>
<thead>
<tr>
<th>Index/Subtest</th>
<th>Composite/ Scaled Score</th>
<th>Verbal Comprehension</th>
<th>114</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comprehension</td>
<td>15</td>
<td>Perceptual Reasoning</td>
<td>125</td>
</tr>
<tr>
<td>Similarities</td>
<td>13</td>
<td>Block Design (W)</td>
<td>11</td>
</tr>
<tr>
<td>Vocabulary (W)</td>
<td>10</td>
<td>Matrix Reasoning (S)</td>
<td>17</td>
</tr>
<tr>
<td>Working Memory</td>
<td>80</td>
<td>Picture Concepts</td>
<td>14</td>
</tr>
<tr>
<td>Digit Span (W)</td>
<td>6</td>
<td>Processing Speed</td>
<td>121</td>
</tr>
<tr>
<td>Letter-Number Sequencing (W)</td>
<td>7</td>
<td>Coding</td>
<td>12</td>
</tr>
<tr>
<td>(Arithmetic)</td>
<td></td>
<td>Symbol Search</td>
<td>15</td>
</tr>
</tbody>
</table>

Full Scale IQ = 115; General Ability Index = 123

Cognitive Strengths and Weaknesses

<table>
<thead>
<tr>
<th>Index Comparisons</th>
<th>Index Score 1</th>
<th>Index Score 2</th>
<th>Diff.</th>
<th>Critical Value (0.05)</th>
<th>Sig. Diff.? Y / N</th>
<th>Base Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCI - PRI</td>
<td>114</td>
<td>125</td>
<td>-11</td>
<td>10.18</td>
<td>Y</td>
<td>22.3%</td>
</tr>
<tr>
<td>VCI - WMI</td>
<td>114</td>
<td>80</td>
<td>34</td>
<td>10.18</td>
<td>Y</td>
<td>1.1%</td>
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<tr>
<td>VCI - PSI</td>
<td>114</td>
<td>121</td>
<td>-7</td>
<td>11.75</td>
<td>N</td>
<td>36.2%</td>
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<tr>
<td>PRI - WMI</td>
<td>125</td>
<td>80</td>
<td>45</td>
<td>11.00</td>
<td>Y</td>
<td>0.4%</td>
</tr>
<tr>
<td>PRI - PSI</td>
<td>125</td>
<td>121</td>
<td>4</td>
<td>12.47</td>
<td>N</td>
<td>40.9%</td>
</tr>
<tr>
<td>WMI - PSI</td>
<td>80</td>
<td>121</td>
<td>-41</td>
<td>12.47</td>
<td>Y</td>
<td>0.8%</td>
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<tr>
<td>FSIQ - GAI</td>
<td>115</td>
<td>123</td>
<td>-8</td>
<td>3.73</td>
<td>Y</td>
<td>7.9%</td>
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</table>

WIAT-III

WIAT-III Composite Score Summary

<table>
<thead>
<tr>
<th>Composite</th>
<th>Standard Score</th>
<th>95% Conf. Interval</th>
<th>%ile Rank</th>
<th>Qualitative Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Reading</td>
<td>82</td>
<td>77–87</td>
<td>12</td>
<td>Below Average</td>
</tr>
<tr>
<td>Basic Reading</td>
<td>82</td>
<td>78–86</td>
<td>12</td>
<td>Below Average</td>
</tr>
<tr>
<td>Reading Comp. and Fluency</td>
<td>86</td>
<td>78–94</td>
<td>18</td>
<td>Average</td>
</tr>
<tr>
<td>Written Expression</td>
<td>81</td>
<td>74–88</td>
<td>10</td>
<td>Below Average</td>
</tr>
<tr>
<td>Mathematics</td>
<td>84</td>
<td>78–90</td>
<td>14</td>
<td>Below Average</td>
</tr>
<tr>
<td>Math Fluency</td>
<td>85</td>
<td>78–92</td>
<td>16</td>
<td>Average</td>
</tr>
</tbody>
</table>

Selected Subtest Scores

<table>
<thead>
<tr>
<th>Subtest</th>
<th>Standard Score</th>
<th>95% Conf. Interval</th>
<th>%ile Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Problem Solving</td>
<td>92</td>
<td>84–100</td>
<td>30</td>
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<tr>
<td>Numerical Operations</td>
<td>79</td>
<td>73–85</td>
<td>8</td>
</tr>
<tr>
<td>Math Fluency Addition</td>
<td>87</td>
<td>76–98</td>
<td>19</td>
</tr>
<tr>
<td>Math Fluency Subtraction</td>
<td>86</td>
<td>76–96</td>
<td>18</td>
</tr>
<tr>
<td>Math Fluency Multiplication</td>
<td>86</td>
<td>76–96</td>
<td>18</td>
</tr>
</tbody>
</table>
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Under-Achievement in Math?

<table>
<thead>
<tr>
<th>Subtest</th>
<th>Predicted Score</th>
<th>Actual Score</th>
<th>Diff.</th>
<th>Critical Value</th>
<th>Sign. Diff.?</th>
<th>Base Rate</th>
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</thead>
<tbody>
<tr>
<td>Math Problem Solving</td>
<td>114</td>
<td>92</td>
<td>22</td>
<td>9.48</td>
<td>Y</td>
<td>&lt;5%</td>
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<tr>
<td>Numerical Operations</td>
<td>111</td>
<td>79</td>
<td>32</td>
<td>8.29</td>
<td>Y</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>Math Fluency Addition</td>
<td>107</td>
<td>87</td>
<td>20</td>
<td>11.54</td>
<td>Y</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>Math Fluency Subtraction</td>
<td>108</td>
<td>86</td>
<td>22</td>
<td>10.40</td>
<td>Y</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>Math Fluency Multiplication</td>
<td>106</td>
<td>86</td>
<td>20</td>
<td>9.42</td>
<td>Y</td>
<td>&lt;10%</td>
</tr>
</tbody>
</table>

Ability: WISC-IV GAI = 123

Does this info give us enough data to make a decision?
What should we do now?
Are there any other patterns we should look for?
What may be driving or contributing to the math issues?
  - What tests would we use to investigate this?

So…what now?

• Does this info give us enough data to make a decision?
• What should we do now?
• Are there any other patterns we should look for?
• What may be driving or contributing to the math issues?
  - What tests would we use to investigate this?

NEPSY-II additions: Inhibition and AS

<table>
<thead>
<tr>
<th>NEPSY-II</th>
<th>Scaled Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inhibition: Naming Combined Score</td>
<td>9</td>
</tr>
<tr>
<td>Inhibition: Inhibition Combined Score</td>
<td>6</td>
</tr>
<tr>
<td>Inhibition: Switching Combined Score</td>
<td>3</td>
</tr>
<tr>
<td>Inhibition: INI vs. INS Contrast Score</td>
<td>4</td>
</tr>
<tr>
<td>Animal Sorting: Combined Score</td>
<td>7</td>
</tr>
</tbody>
</table>

Behavior Rating

D-REF

- Clinically significant Core Index Scores:
  - Behavioral Functioning T=62
  - Executive Functioning T=65
  - Total Composite T=61

- Clinically significant Clinical Index Scores:
  - Attention/Working Memory T=66
  - Activity Level/Impulse Control T=61
  - Abstract Thinking/Problem Solving T=63

Starts to get me thinking…
Attention…Working Memory?
...now bring in the NEPSY-II
Does Steven fit this description?

- Procedural Math LD (Geary, 2004)
  - Adequate quantitative and quantity-symbol knowledge
  - Weak arithmetic strategies and algorithm use with many calculation errors
  - Limitations in verbal working memory and conceptual knowledge
  - Slow processing speed
  - May involve attention and executive dysfunction
  - Often associated with ADHD-Inattentive Type

Eligibility and Need For Direct Specialized Instruction

Determining the Existence of a Specific Learning Disability

Specific Learning Disability

IDEA 2004

...a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations, ... (§ 300.8)

The child does not achieve adequately for the child’s age or to meet State-approved grade-level standards in one or more of the following areas, when provided with learning experiences and instruction appropriate for the child’s age or State-approved grade-level standards: § 300.309 (a) (1))

- Oral Expression
- Written Expression
- Basic Reading Skills
- Mathematics Calculation
- Listening Comprehension
- Reading Comprehension
- Reading Fluency Skills
- Mathematics Problem-Solving

Specific Learning Disability

IDEA 2004

The child does not make sufficient progress to meet age or State approved grade-level standards in one or more of the areas identified in paragraph (a)(1) of this section when using a process based on the child’s response to scientific, research-based intervention; or

§ 300.309 (a) (2) (i)

Pattern of Strengths and Weaknesses

the child exhibits a pattern of strengths and weaknesses in performance, achievement, or both, relative to age, State-approved grade level standards or intellectual development.

(§ 300.309 (a) (2) (ii)).
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Steven: Pattern of Strengths and Weaknesses

Processing Strength
WISC–IV Verbal Comprehension Index
SS = 114

A. Discrepant?
Yes

B. Discrepant?
Yes

Achievement Weakness
WIAT–III Numerical Operations
SS = 79

Processing Weakness
WISC–IV Working Memory Index
SS = 80

Pattern of Strengths and Weaknesses Analysis

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Relative Strength Score</th>
<th>Relative Weakness Score</th>
<th>DIF</th>
<th>Critical Value</th>
<th>Sign</th>
<th>Diff. Y/N</th>
<th>Supports SLD hypothesis?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Processing Strength / Achievement Weakness</td>
<td>114 (VCI)</td>
<td>79 (WMI)</td>
<td>35</td>
<td>8.82</td>
<td>Y</td>
<td>Y</td>
<td>Yes</td>
</tr>
<tr>
<td>B Processing Strength / Processing Weakness</td>
<td>114 (VCI)</td>
<td>80 (WMI)</td>
<td>54</td>
<td>10.18</td>
<td>Y</td>
<td>Y</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The PSW model is intended to help practitioners generate hypotheses regarding clinical diagnoses. This analysis should always be used within a comprehensive evaluation that incorporates multiple sources of information.

Why Did Steven Struggle with Basic Math Operations?

Written Calculation Requires:
- Counting fluency.
- Quantitative working memory.
- Phonological and orthographic loops in working memory.
- Numerical writing.
- Oral and written fluency in retrieving math facts from memory.
- Spatial alignment of numbers on the page.
- Application of algorithms—sequential computation steps.
- Place value concept.
- Part-whole concept.
- Executive functions for mental set switching and self-monitoring.

Summary of Findings

- Steven's achievement in reading, mathematics, and writing is a weakness relative to his higher-order reasoning abilities.
- His achievement is adversely affected by weaknesses in registration and manipulation of auditory information.
- His achievement in basic mathematics skills is adversely affected by weaknesses in retrieving math facts from long-term memory.

Recommendations

- Steven's performance in academic areas was affected adversely by weaknesses in registration and manipulation of auditory information. Therefore, we recommend assessment of his auditory processing abilities.
- Further assessment of attention and executive functioning will provide information on his strengths and needs in these domains. Information on cognitive efficiency will be helpful for instructional planning.

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Recommendations

- Interventions for math should focus on improving accuracy and fluency of basic facts.
- A calculator may be helpful when Steven is completing mathematics reasoning tasks.

Improving Automaticity

To help Steven to commit math facts to automatic recall, allow him to practice them with different combinations of input (looking or listening) and output (writing or saying).

- Present the math fact visually. Steven looks at the math fact and then writes the sum, difference, product, or quotient.
- Present the math fact visually. Steven looks at the math fact and then says the sum, difference, product, or quotient.
- Present the math fact audibly. Steven listens to the math fact and then writes the sum, difference, product, or quotient in writing.
- Present the math fact audibly. Steven listens to the math fact and then says the sum, difference, product, or quotient in writing. (Berninger, 2007).

Short-term Memory Interventions

Most interventions to improve short-term memory involve rehearsal training.

Rehearsal Strategies

- Say the material over and over to oneself.
- Engage in serial repetition. This allows information to be maintained in WM for longer periods of time, thus enhancing short-term recall. Elaborative rehearsal also facilitates long-term storage.

Verbal Working Memory Interventions

Chunking

- Pairing, clustering, grouping, or association of different items into units that are processed and remembered as a whole. This facilitates short-term retention and consolidation into long-term storage.

Paraphrasing

- A strategy that builds on both rehearsal and chunking. Students restate information in their own words. This requires that they reorganize and condense a large amount of linguistic information into smaller, well-integrated, and more personally meaningful units.

Executive Working Memory Interventions

Dual Encoding

- Strategies utilizing concurrent visual and verbal encoding.
- Some dual encoding occurs naturally (e.g., number naming).
- In the classroom, visual and verbal materials should be utilized.
Executive Working Memory Interventions

Organizational Strategies
- Fitting existing information into an organized structure (semantic category).
- Structuring and organizing information reduces the processing load on WM, thereby allowing more efficient encoding.
- Organizing information involves rehearsal and the processing of information at a deeper level.

References


Thanks for attending this webinar!!

For customers in the USA
1-800-627-7271
www.pearsonclinical.com

For customers in Canada
1-866-335-8418
www.pearsonassess.ca

Adam Scheller, Ph.D.
Pearson Training and Consultation
adam.scheller@pearson.com


